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On the numerical simulation of vortex-induced vibrations of oscillating conductors

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Abstract

Aeolian vibrations for electrical overhead transmission line conductors have been investigated for many decades. Special dampers, e.g., Stockbridge dampers or spacer dampers, are mounted on the conductors to suppress these vibrations, which may otherwise lead to the fatigue failure at the points of high strain values. Simulations are routinely carried out in order to estimate the vibration levels, to determine the need of dampers, and to optimize their locations and the impedances. The energy balance principle (EBP) is well established for estimating the vibration amplitudes, and hence, the strain levels in the transmission line conductors. Besides the parameters of the conductor and of the dampers, the aerodynamic forces acting on the vibrating conductor are the main input data required for the energy balance. For the wind power input, researchers still depend on the experimental data of drag and lift forces of a vibrating cylinder obtained from wind tunnel testing. In case of the bundled conductors, many combinations regarding the number of conductors, spacing of the conductors as well as their orientations are possible, which make wind tunnel tests very expensive and formidable. It may be useful to replace the wind tunnel tests by numerical simulations, as far as possible. However, it is indispensable to validate the numerical results first, for at least some special cases, so that they can be used with confidence in the general case. The present paper is a first step towards obtaining the wind power inputs for different configurations of bundled conductors. In the current work, the flow around a vibrating conductor is simulated with the finite-volume method, by considering it as a circular cylinder. The two-dimensional Navier–Stokes equations are solved first. The drag and the lift forces are then calculated by integrating the pressure and the shear values on the boundary of the cylinder, which ultimately cause the impartation of wind power. The numerically obtained wind power input is then compared with that obtained by different researchers in wind tunnel tests. A very good match between the experimental and the numerical values of wind power input is found.

1. Introduction

Among the different mechanical vibration phenomena in high-voltage electrical overhead transmission lines, wind-excited oscillations of the conductors constitute a major issue. The most common of these vibrations are those generated by vortex shedding in the frequency range of approximately 3–50 Hz, corresponding to wind speeds of 1–7 m/s,

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see EPRI (1979). Although such vibrations are barely perceptible due to their low amplitudes (less than a conductor diameter), they are, however, extremely important since they may lead to conductor fatigue at the positions of high-strain concentration (i.e., suspension clamps). Mathematical models are therefore necessary for the computation of these vibrations in order to evaluate the risk of potential damage to the conductor, as well as for studying the efficiency of the damping measures. Although it is a complicated problem of fluid-structure interaction, the simulations are normally carried out with mechanical-mathematical models using the energy balance principle (EBP) as shown in Hagedorn (1980, 1982) and Verma and Hagedorn (2005).

In the EBP procedure, an eigenvalue problem is solved first to obtain the natural frequencies and the corresponding mode shapes of the single-conductor transmission line equipped with a number of Stockbridge dampers, or of the bundled conductors with spacer dampers. In the next step, actual vibration amplitudes are estimated by scaling these mode shapes, equating the power input from the wind to the power dissipated by self-damping of the conductor and that by the Stockbridge dampers (in the case of single conductor transmission lines) or that by the spacer dampers (in the case of bundled conductors):

$$P_W = P_D + P_C, \tag{1}$$

where P_W is the wind power input, P_C is the power dissipation by the conductor's self damping, and P_D is the power dissipated by the Stockbridge or the spacer dampers. The wind power input P_W is normally computed from the experimentally obtained aerodynamic forces from wind tunnel experiments. These are carried out by measuring the drag and lift forces on the oscillating cylinders, flexible rods or cables under uniform laminar cross-flow in the wind tunnel [as shown in Diana and Falco (1971), Rawlins (1983), Bishop and Hassan (1964) and Belloli et al. (2003)]. The present paper aims to obtain the same wind power input numerically, which could be helpful in the future to replace the expensive wind tunnel tests in more complex situations.

The numerical solution of the fluid problem itself is usually a nontrivial task, often requiring the use of complex (nonlinear) models and high numerical resolution in order to achieve the necessary modelling quality and the discretization accuracy.

Considering the transmission line span, the variations of wind velocity and turbulence along the length of the conductor are neglected in the EBM and, for the purpose of obtaining the wind power input, the problem is treated as two-dimensional. In the present paper, the two-dimensional problem of a harmonically oscillating cylinder in a laminar cross-flow is simulated using the finite-volume method, and the flow problem is accordingly solved for time-dependent boundary conditions. In the computations, the grid is deformed according to the motion of the structure (i.e., the cylinder). With the help of an *arbitrary Lagrangian-Eulerian* formulation (ALE), the fluxes through the surfaces of the control volumes (CVs) are corrected using a "space conservation law" (SCL), that requires modifications within the flow solver. Here, conservation properties must be guaranteed (Lesoinne and Farhat, 1996), e.g., through the use of Lagrangian multipliers as proposed by Le Tallec (2001) or load projection algorithms (Cebral and Löhner, 1997).

The emphasis is mainly on the accurate computation of the aerodynamic forces acting on the moving cylinder. In particular, for any given wind speed the time-averaged power of the aerodynamic forces acting on the moving cylinder is computed, since this is measured in the wind tunnel experiments and used in the computations of the vibrations of overhead transmission lines. Moreover, this type of wind tunnel test data is not available for bundled conductors, because of the many different possible configurations of the conductors in a conductor bundle and also because of the complexity of their oscillations. Though the wind power input to a single vibrating conductor is the objective of the present paper, it will also pave the way for computing the wind power input in bundled conductors. An essential task in introducing the numerical simulation in this field is the experimental validation of the reliability of the numerical data, which is shown in Section 4.

2. Wind power input

A vibrating conductor receives energy from the wind and its amplitude rises to a point at which the vibration energy dissipated in the conductor, at the boundaries and eventually in the dampers balances the energy imparted by the wind. Bate (1930) developed an expression for wind energy input and conducted experiments on conductor dissipation. The wind power transferred from the wind to the conductors may be defined in terms of specific power $\tilde{P}_W$, so that it is independent of the model and of the motion characteristics—i.e., vibration frequency, model geometry (EPRI, 1979; Diana and Falco, 1971; Rawlins, 1983; Brika and Laneville, 1995). The wind power input for a conductor of diameter D is expressed in the general form:

$$P_W = \tilde{P}_W f^3 D^4 L, \tag{2}$$

where P_W is the mechanical power, L is the span length and f is the frequency of the vibration of the conductor. The specific power $\tilde{P}_W$ has been measured experimentally by different researchers and results are shown in Fig. 6, to be discussed later.

2.1. Numerically obtaining the wind power input

In the present numerical analysis, the wind power input on a harmonically vibrating cylinder of unit length is calculated by integrating the product of the lift force and the velocity of the cylinder. The cylinder vibrates harmonically with a given frequency and amplitude, and laminar wind in cross-flow with a given velocity impinges on the cylinder. After getting the mechanical power from the wind $P_{W,\mathrm{num}}$, the specific power $\tilde{P}_{W,\mathrm{num}}$ is calculated by normalizing it with the factor f^3D^4L, according to Eq. (2).

It is known from field observations that the amplitude of the Aeolian vibrations rarely exceeds one conductor diameter. Wind tunnel testings provide an explanation for this property of Aeolian vibrations. Different researchers have determined that for a vibration amplitude of up to approximately $0.7D$, the wind imparts energy to the vibrating cylinder. Hence, the wind power input is positive. For higher amplitudes than this, the cylinder starts giving energy back to the wind, as can be seen from the downfall of the experimental curves in Fig. 6. This is equivalent of having damping, and hence the vibration amplitude starts decreasing and normally does not go beyond this limit of approximately $0.7D$.

Because a transmission line has a very dense spectrum of natural frequencies, in the EBP it is assumed that approximately every frequency is a natural frequency of the transmission line system. For the sake of simplicity, it can thus be safely assumed that for the whole wind speed range (i.e., 1–7 m/s) the conductor will oscillate in resonance. In other words, the lock-in phenomenon will always take place, i.e., $f = f_s$ in Eq. (2). This is also the reason for always taking the wind power input data corresponding to the lock-in frequency into consideration in the energy balance approach. Consequently, in the present analysis the vibration amplitudes of the cylinder are kept between $0.1D$ and $1.0D$ and the vibration frequency is equal to the Strouhal frequency f_s.

The transverse motion of the cylinder is defined a priori, so that the problem is one of the time-dependent boundary conditions,

$$y(t) = \hat{A}\,\sin(2\pi f_s t), \tag{3}$$

which gives the velocity of vibration in the time domain as

$$\dot{y}(t) = 2\hat{A}\pi f_s\,\cos(2\pi f_s t). \tag{4}$$

Here, y is the transverse displacement of the cylinder at time t, with $\hat{A}$ as the vibration amplitude and f_s is the Strouhal frequency for the corresponding wind velocity, which is given by

$$f_s = \frac{\mathrm{St}\,U}{D}, \tag{5}$$

the wind speed is U, D is the conductor diameter and St is the Strouhal number, which varies from 0.18 to 0.2 for circular cylinders. Considering the appropriate value of the Strouhal number is important in the present analysis, as discussed in Section 4.

3. Computational fluid dynamics

The motion of the obstacle (i.e., of the cylinder) causes a change in the fluid domain and consequently in the fluid behavior. Therefore, the motion of the fluid grid affects the numerical solution of the Navier–Stokes equations. Applying the finite volume method (FVM) to the fluid part, a modification within the flow solver is required by using a space conservation law SCL (Demirdžić and Perić, 1990). This method was proposed to correct the continuity equation in which the grid velocity has to be taken into account in the mass fluxes through the surfaces of the CVs and in the mass source due to the change of the volume of the CVs.

3.1. Governing equations

The flow of an incompressible Newtonian fluid is described by the Navier–Stokes equations. Problems concerning fluid-structure interaction lead to motion of the boundaries of the fluid domain. Thus, it is usually convenient to use a numerical grid varying in time. Here, the general transport equations are extended for moving grids (Thomas and

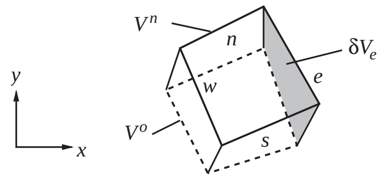

Fig. 1. Moving two-dimensional control volume with swept volume δV_e.

Lombard, 1979) yielding the balance of momentum

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{V(t)} \rho v_i \,\mathrm{d}V + \int_S [\rho v_i (v_j - v_j^g) - t_{ij}] n_j \,\mathrm{d}S = \int_V \rho f_i \,\mathrm{d}V, \tag{6}$$

and the mass conservation

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{V(t)} \rho \,\mathrm{d}V + \int_S \rho (v_j - v_j^g) n_j \,\mathrm{d}S = 0 \tag{7}$$

in standard notation (Schäfer, 1999), and the grid velocity v_j^g as an additional variable. To ensure the conservation principle for the motion of the CVs, the SCL can be derived:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{V(t)} \mathrm{d}V - \int_S v_j^g n_j \,\mathrm{d}S = 0. \tag{8}$$

The change of the volume of the CVs can be computed in different ways, as proposed in Demirdžić and Perić (1990). The influence of the grid velocity of a CV as described in the above equation is considered as follows:

$$\int_S v_j^g n_j \,\mathrm{d}S = \sum_c (v_j^g n_j S)_i = \sum_c \frac{\delta V_c}{\Delta t}, \quad \forall c = n, e, s, w, \tag{9}$$

where δV_c denotes the swept volume within the time step Δt for the face c of the CV (see Fig. 1 for the two-dimensional case).

3.2. Solution procedure

The finite volume program FASTEST INVENT Computing GmbH (1994) is used for the solution of the fluid dynamics problem. To discretize the Navier–Stokes equations, a second-order accurate approximation for the integrals by the midpoint rule is employed [see, e.g., Durst and Schäfer (1996) or Ferziger and Perić (1997)]. The CDS method (central differencing scheme) is applied for the interpolation of the values in the CV center to the center of the CV faces, as well as the computation of the gradients for the diffusive fluxes. A Crank–Nicholson scheme with a second order of accuracy is used for the time integration of the CFD part.

A pressure-correction algorithm is applied to solve the system of equations arising from the discretization for each time step. In FASTEST, a SIMPLE-algorithm (Semi-Implicit Method for Pressure Linked Equations) is used (Patankar and Spalding, 1972). The capabilities of the considered approaches with respect to accuracy and efficiency were investigated by representative test cases involving different types of mechanical and thermal coupling. The results are based on methodologies described in more detail in Meynen and Schäfer (1999), Schäfer et al. (2000) and Meynen et al. (2000).

For the discretization of the fluid domain, a block-structured grid is used. Block-structured grids, which are globally unstructured but locally structured, can be viewed as a compromise between the high geometrical flexibility of fully unstructured grids and the high numerical efficiency achieved on globally structured grids. A parallel version of FASTEST is used for the simulation to reduce the overall computation time. The parallelization of the domain is achieved by a grid-partitioning technique based on the block-structured grids. Depending on the number of processors, the block structure suggested by the geometry is restructured such that the resulting subdomains can be assigned suitably to the individual processors. Here, load balance must be guaranteed to avoid a loss of parallel efficiency. The parallel performance of the finite-volume program was tested in Durst and Schäfer (1996), where more detailed information can be found.

3.3. Grid modification and boundary conditions

For moving rigid bodies, no discretization of the structure is necessary. The fluid mesh is kept attached to material points $x_i^{\text{struc}}|_{\text{wall}}$ on the surface of the obstacle. No gap or overlapping between the fluid and the structure is allowed on the coupling interface. This requires identical coordinates of the structure x_i^{struc} and the fluid grid x_i^g:

$$x_i^{\text{struc}}|_{\text{wall}} = x_i^g|_{\text{wall}}. \tag{10}$$

Hence, the new position x_i^g of the fluid nodes on the moving wall is directly determined from the degrees of freedom of the rigid body x_i. To regenerate the fluid mesh after a motion of the structure, the internal grid points of the fluid domain are moved by a linear distribution of the displacements of the points on the boundaries.

For viscous fluids, the velocity at a wall is equal to the velocity of the wall due to the no-slip condition. Thus, the velocity of the fluid grid u_i^g is

$$\dot{x}_i|_{\text{wall}} = u_i^g|_{\text{wall}}. \tag{11}$$

As already mentioned above, the cylinder oscillates harmonically with the lock-in frequency, so that the displacement as well as the velocity can be prescribed in a sinusoidal manner, according to Eqs. (4) and (5).

4. Numerical results

For numerical investigations a two-dimensional discretization is applied to compute the reaction of the fluid flow due to the prescribed cylinder motion. The block structured finite-volume grid of the fluid domain is shown in Fig. 2. On the left side, a uniform inflow velocity of 0.1 m/s is prescribed, while at the outflow on the right side a zero gradient condition is assumed. The height of the fluid domain is 2 m (30D) and the inflow zone on the left side of the cable is 4 m (60D). A long outflow region of 45 m (675D) is used to avoid any influence of the outflow boundary conditions. The cable diameter is chosen as $D = 0.0667$ m. Symmetric boundary conditions are applied above and below. Fig. 3 shows a circular zone of fine discretization around the cylinder to achieve high accuracy for the lift and drag forces. Furthermore, these blocks are moved rigidly with the cylinder. Any grid deformation is deferred to the outer region.

The standard material parameters for air are used for the fluid; density $\rho = 1.23\,\text{kg/m}^3$ and viscosity $\mu = 1.78 \times 10^{-5}\,\text{kg/m s}$. Consequently, the inflow velocity and the cylinder diameter yield a Reynolds number of

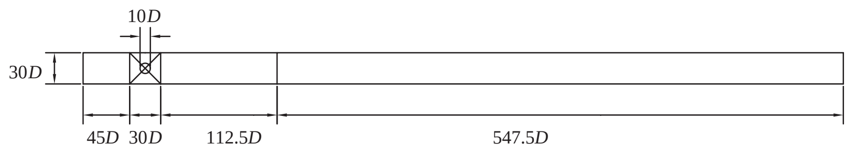

Fig. 2. Block structured discretization of the numerical domain.

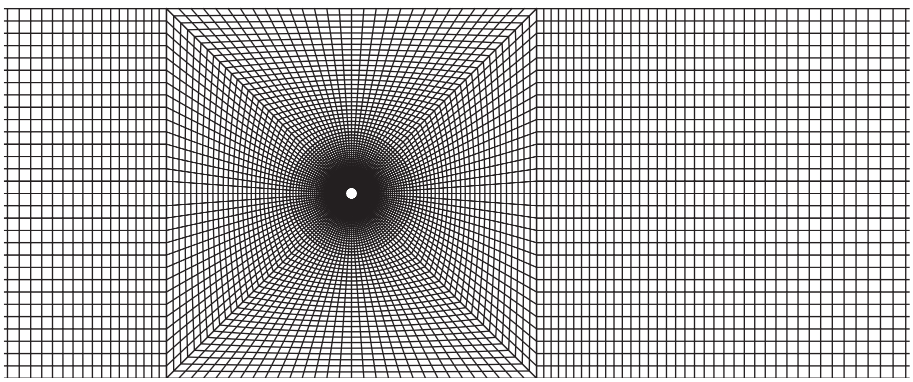

Fig. 3. Structured finite-volume grid.

Re ≈ 460, which is in the laminar transient range. For the discretization of the diffusive terms, a central differencing scheme (CDS) is used together with the Crank–Nicholson time-stepping scheme. Hence, a spatial and temporal discretization of second order is obtained. A time step size of $\Delta t = 0.01$ s is chosen. Fig. 4 shows the streamlines and rotation of the velocity field. Here, only the part around the conductor is shown, rather than the whole numerical domain.

The frequency of the vibration of the cylinder is obtained from Eq. (5) by considering the appropriate value of the Strouhal number, which is usually between 0.18 to 0.20 for the cylinder. We emphasize that a small variation in the Strouhal number may cause a significant change in the flow behavior around the cylinder. This results in different wind power input values for the same amplitude of vibration. As already explained earlier, because of the dense frequency spectrum of the transmission line system, it can be safely assumed that for all wind speeds lock-in phenomena will take place. Because of this fact, a value of the Strouhal number should be considered in Eq. (5) for which the vibrating cylinder gets the maximum power input from the wind. We repeated the computations for different Strouhal numbers and different amplitudes, and found that instead of St = 0.20, St = 0.194 to 0.198 give higher wind power inputs, as can be seen for example in Fig. 5. These values of Strouhal number also correspond to the experimentally obtained value for Re ≈ 460, as shown in Zdravkovich (1997, p. 121). Hence, in the current analysis, different frequencies corresponding to the maximum wind power inputs have been used for the cylinder vibrations.

The wind power input $P_{W,\text{num}}$ is derived from the lift force F_L. These results are used to compare the numerical simulation with the experimental data, as described earlier in the paper. The simulation for a normalized amplitude with respect to the conductor diameter ($\hat{A}/D$ ratio) coincides very well with the experiments, as can be seen from Fig. 6. Numerical tests confirm the correct overall behavior of the simulations, which shows that the current analysis can now be extended to multi-cylinder problems, where it should be a useful tool for bundled conductor transmission line analysis.

5. Conclusion

In the present paper, the two-dimensional problem of a harmonically oscillating cylinder in steady laminar cross-flow is solved by numerically integrating the Navier–Stokes equations. A lock-in is assumed for all wind speeds due to the very dense spectrum of natural frequencies of the conductors of a transmission line. Therefore, the conductor will

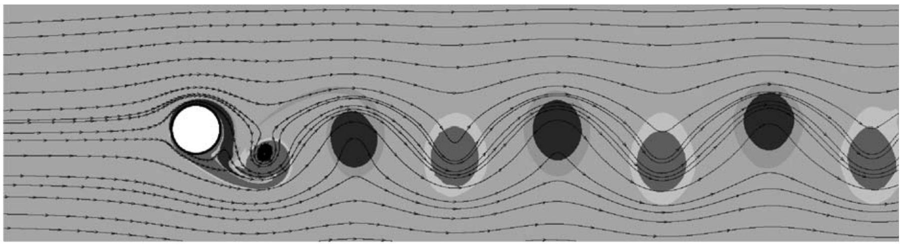

Fig. 4. Streamlines and rotation of the velocity field.

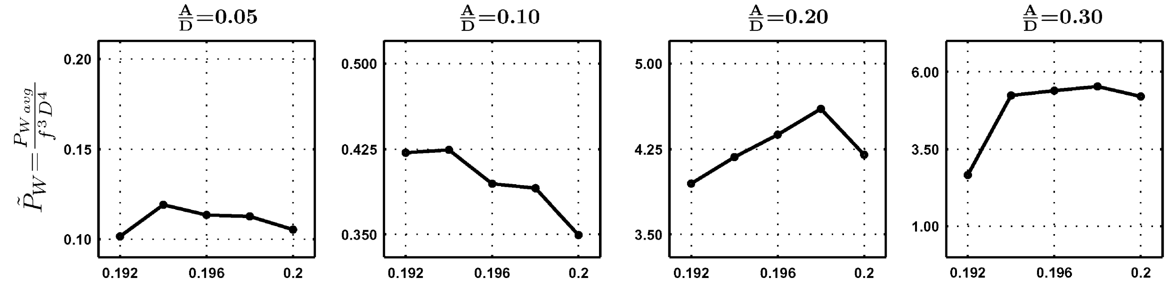

Fig. 5. Specific wind power input with frequencies corresponding to different St-values.

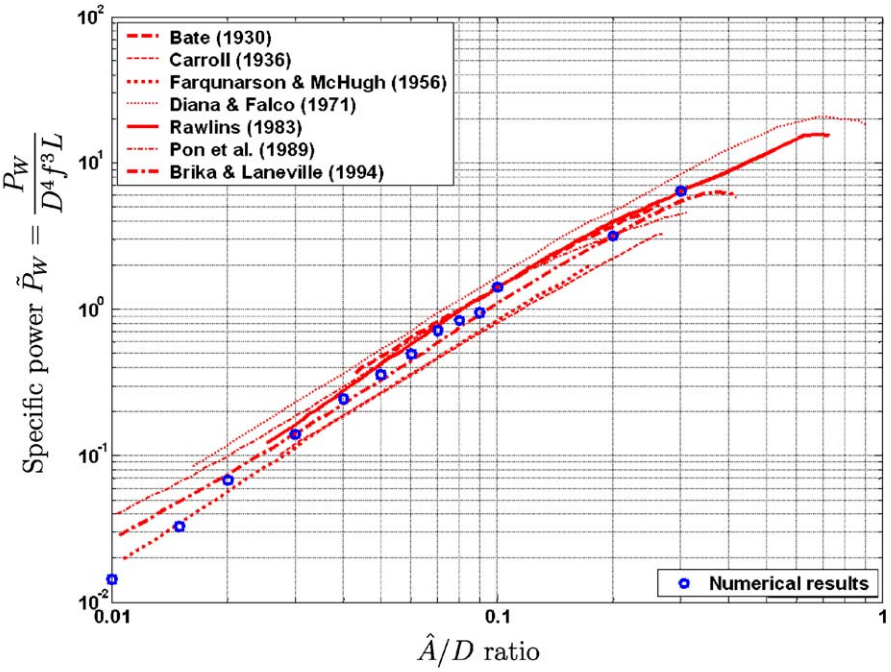

Fig. 6. Specific power input, comparison between simulation and experiments [see Belloli et al. (2003)].

oscillate in resonance. The numerical results obtained with the above assumptions were compared with wind tunnel experiments, and a very good agreement was found with the experimental data. Since the numerical simulation is now validated for a single conductor, it may in the future also be used for the investigation of bundled conductors with respect to the influence of, for example, variation of the number of cables, position and orientation.

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